Physics 4A
Chapter 4: Kinematics in Two Dimensions

"There is nothing either good or bad but thinking makes it so." – William Shakespeare

"It's not what happens to you that determines how far you will go in life; it is how you handle what happens to you." – Zig Ziglar

Reading: pages 80 – 92; 96 – 98

Outline:

⇒ motion in two dimensions
- position and displacement
- average and instantaneous velocity
- average and instantaneous acceleration

⇒ projectile motion
- equations of projectile motion
- example problems

⇒ relative motion (read on your own)

⇒ centripetal acceleration

Problem Solving

The problems of this chapter deal mainly with the definitions of average and instantaneous velocity and acceleration, with projectile motion, and with centripetal acceleration.

To calculate the average velocity, you need to know the position at the beginning and end of a time interval. To calculate the velocity, you need to know the position as a function of time. To calculate the average acceleration, you need to know the velocity at the beginning and end of a time interval. To calculate the acceleration, you need to know the velocity as a function of time.

When you read a projectile motion problem, you should be able to identify two events, just as you did for one-dimensional problems. Take the time to be 0 for one of them, the launching of the projectile, for example. Take the y axis to be vertically upward and the x axis to be horizontal in the plane of the motion. The coordinates and velocity components are x_0, y_0, v_{0x}, and v_{0y} for the event at time 0. Let t be the time of the other event. The coordinates and velocity components are x, y, v_x, and v_y for that event. Identify the known and unknown quantities, then solve for the unknown quantities.

All centripetal acceleration problems are solved using $a = v^2/r$. This equation contains three quantities. Two must be given, either directly or indirectly. Remember that the acceleration vector points toward the center of the circle if the speed is constant. Sometimes the period T of the motion is given. Remember that $v = 2\pi r/T$, where r is the radius of the circular orbit. You can use this expression to eliminate v in favor of r or r in favor of v in the expression for the centripetal acceleration.

GENERAL PRINCIPLES

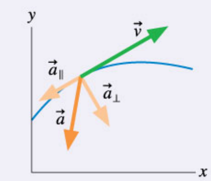
The **instantaneous velocity**

$$\vec{v} = d\vec{r}/dt$$

is a vector tangent to the trajectory.

The **instantaneous acceleration** is

$$\vec{a} = d\vec{v}/dt$$



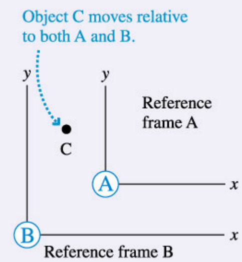
$\vec{a}_{\parallel}$, the component of $\vec{a}$ parallel to $\vec{v}$, is responsible for change of *speed*. $\vec{a}_{\perp}$, the component of $\vec{a}$ perpendicular to $\vec{v}$, is responsible for change of *direction*.

Relative Motion

If object C moves relative to reference frame A with velocity $\vec{v}_{CA}$, then it moves relative to a different reference frame B with velocity

$$\vec{v}_{CB} = \vec{v}_{CA} + \vec{v}_{AB}$$

where $\vec{v}_{AB}$ is the velocity of A relative to B. This is the Galilean transformation of velocity.



IMPORTANT CONCEPTS

Uniform Circular Motion

Angular velocity $\omega = d\theta/dt$.

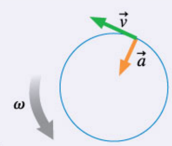
v_t and ω are constant:

$$v_t = \omega r$$

The centripetal acceleration points toward the center of the circle:

$$a = \frac{v^2}{r} = \omega^2 r$$

It changes the particle's direction but not its speed.



Nonuniform Circular Motion

Angular acceleration $\alpha = d\omega/dt$.

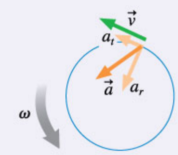
The radial acceleration

$$a_r = \frac{v^2}{r} = \omega^2 r$$

changes the particle's direction. The tangential component

$$a_t = \alpha r$$

changes the particle's speed.



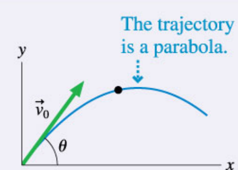
APPLICATIONS

Kinematics in two dimensions

If $\vec{a}$ is constant, then the x- and y-components of motion are independent of each other.

$$x_f = x_i + v_{ix}\,\Delta t + \tfrac{1}{2}a_x(\Delta t)^2$$
$$y_f = y_i + v_{iy}\,\Delta t + \tfrac{1}{2}a_y(\Delta t)^2$$
$$v_{fx} = v_{ix} + a_x\,\Delta t$$
$$v_{fy} = v_{iy} + a_y\,\Delta t$$

Projectile motion is motion under the influence of only gravity.



MODEL Model as a particle launched with speed v_0 at angle θ.

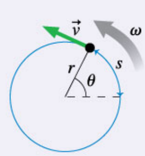
VISUALIZE Use coordinates with the x-axis horizontal and the y-axis vertical.

SOLVE The horizontal motion is uniform with $v_x = v_0 \cos\theta$. The vertical motion is free fall with $a_y = -g$. The x and y kinematic equations have the *same* value for Δt.

Circular motion kinematics

Period $T = \dfrac{2\pi r}{v} = \dfrac{2\pi}{\omega}$

Angular position $\theta = \dfrac{s}{r}$



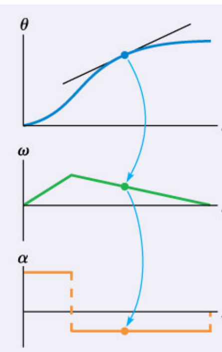
Constant angular acceleration

$$\omega_f = \omega_i + \alpha\,\Delta t$$
$$\theta_f = \theta_i + \omega_i\,\Delta t + \tfrac{1}{2}\alpha(\Delta t)^2$$
$$\omega_f^2 = \omega_i^2 + 2\alpha\,\Delta\theta$$

Circular motion graphs and kinematics are analogous to linear motion with constant acceleration.

Angle, angular velocity, and angular acceleration are related graphically.

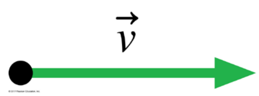
- The angular velocity is the slope of the angular position graph.
- The angular acceleration is the slope of the angular velocity graph.

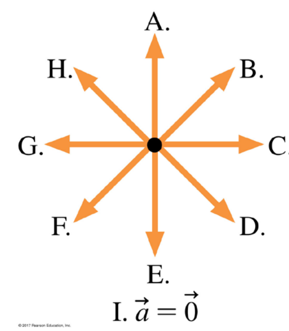


Conceptual Questions and Example Problems from Chapter 4

Conceptual Question 4.5
For a projectile, which of the following quantities are constant during flight: $x, y, r, v_x, v_y, v, a_x, a_y$? Which of these quantities are zero throughout the flight?

Problem 4.4
At this instant, the particle on the left is speeding up and curving upward. What is the direction of its acceleration?





Problem 4.13
A supply plane needs to drop a package of food to scientists working on a glacier in Greenland. The plane flies 100 m above the glacier at a speed of 150 m/s. How far short of the target should it drop the package?

Problem 4.15
In the Olympic shotput event, an athlete throws the shot with an initial speed of 12.0 m/s at a 40.0° angle from the horizontal. The shot leaves her hand at a height of 1.80 m above the ground. How far does the shot travel?

Problem 4.17
A baseball payer friend of yours wants to determine his pitching speed. You have him stand on a ledge and throw the ball horizontally from a distance of 4.0 m above the ground. The ball lands 25 m away. What is his pitching speed?

Problem 4.31
Peregrine falcons are known for their maneuvering ability. In a tight circular turn, a falcon can withstand a centripetal acceleration 1.5 times free-fall acceleration. What is the radius of turn if the falcon is flying at 25 m/s?

Problem 4.47
(a) A projectile is launched with a speed v_0 and angle θ. Derive an expression for the projectile's maximum height. **(b)** A baseball is thrown with a speed of 33.6 m/s. Calculate its height and distance traveled if it is hit at angles of 30.0°, 45.0°, and 60.0°.

Problem 4.53
A 35-g steel ball is held by ceiling-mounted electromagnet 3.5 m above the floor. A compressed-air cannon sits on the floor, 4.0 m to one side of the point directly under the ball. When a button is pressed, the ball drops and, simultaneously, the cannon fires a 25-g plastic ball. The two balls collide 1.0 m above the floor. What was the launch speed of the plastic ball?

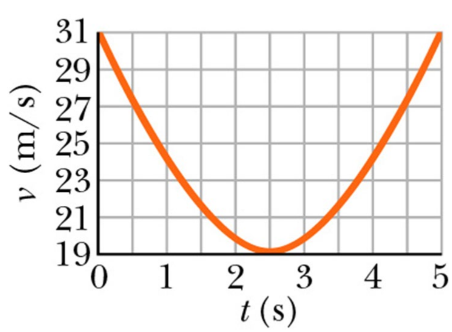
Problem 4.51
A tennis player hits a ball 2.0 m above the ground. The ball leaves his racquet with a speed of 20.0 m/s at an angle of 5.0° above the horizontal. The horizontal distance to the net is 7.0 m, and the net is 1.0 m high. Does the ball clear the net? If so, by how much? If not, by how much does it miss?

Problem 4.A
An electron's position is given by $\vec{r} = (3.00t)\hat{i} - (4.00t^2)\hat{j} + 2.00\hat{k}$, with t in seconds and $\vec{r}$ in meters. **(a)** In unit-vector notation, what is the electron's velocity $\vec{v}(t)$? At t = 2.00 s, what is $\vec{v}$ **(b)** in unit-vector notation and as **(c)** a magnitude and **(d)** an angle relative to the +x-axis.

Problem 4.B
A small ball rolls horizontally off the edge of a tabletop that is 1.20 m high. It strikes the floor at a point 1.52 m horizontally from the table edge. **(a)** How long is the ball in the air? **(b)** What is its speed at the instant it leaves the table?

Problem 4.C
A rocket is fired at a speed of 75.0 m/s from ground level, at an angle of 60.0° above the horizontal. The rocket is fired toward an 11.0 m high wall, which is located 27.0 m away. By how much does the rocket clear the top of the wall?

Problem 4.D
A golf ball is struck at ground level. The speed of the golf ball as a function of the time is shown in the figure below, where t = 0 at the instant the ball is struck.
(a) How far does the golf ball travel horizontally before returning to ground level? **(b)** What is the maximum height above ground level attained by the ball?



Problem 4.E
(a) What is the magnitude of the centripetal acceleration of an object on Earth's equator owing to the rotation of Earth? **(b)** What would the period of rotation of Earth have to be for objects on the equator to have a centripetal acceleration with a magnitude of 9.8 m/s^2?